Welfare Cost of Inflation in Production Networks

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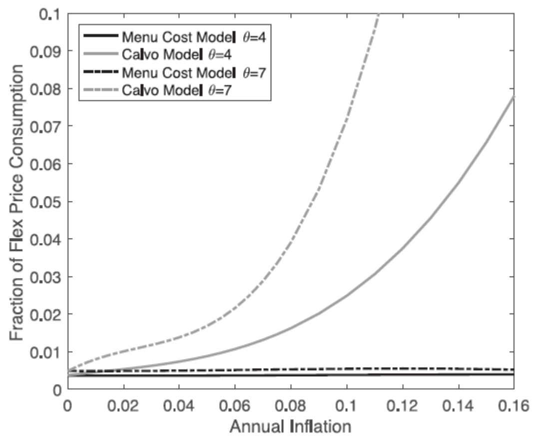
Welfare Cost of Inflation in Standard New Keynesian Models

- At moderate levels of inflation, welfare costs are negligible and flat



Source: Nakamura et al. (2018)

- Current dilemma: Should the Fed stop at 3% or go all the way to 2% (Ball, 2014)?

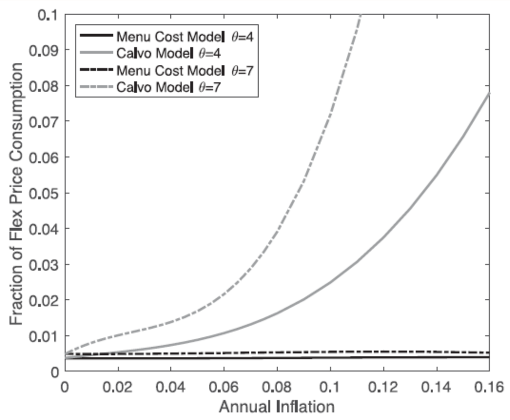
Welfare Cost of Inflation in NK Models with Production Networks

- Standard NK model has one sector and no input-output linkages
- Christiano (2015): Roundabout production amplifies inflation cost
- **This Paper**:
  - *Result 1:* Heterogeneous price stickiness also amplifies the cost of inflation
  - *Result 2:* The two channels interact in a non-trivial way
  - *Result 3:* Together, they amplify the cost of inflation by an order of magnitude

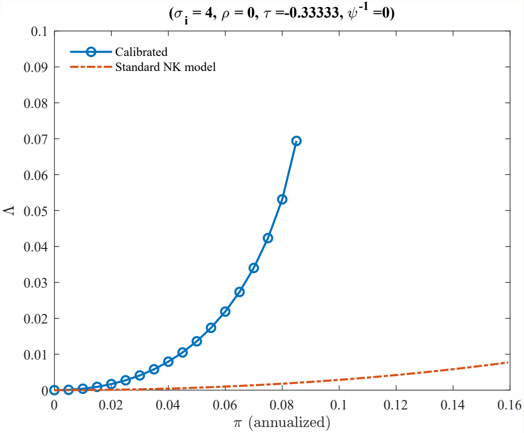
What We Do

- Multi-sector production networks model with heterogeneous price stickiness
- Theoretically, decompose sources of welfare losses from inflation
- Quantitatively, show roles of price stickiness and network structure
    - Using data on US I-O tables and sectoral price stickiness

What We Find: Inflation Is ∼15 Times More Costly with Production Networks



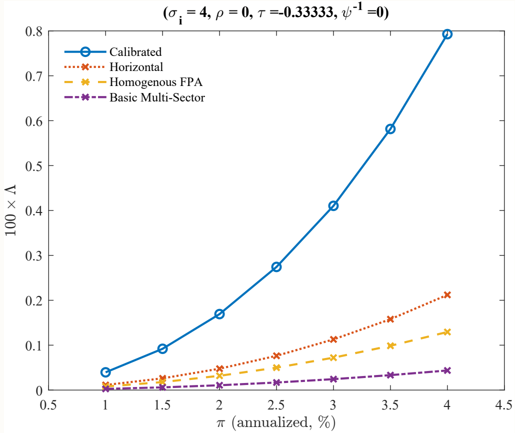
(a) Nakamura et al. (2018)



(b) Cobb-Douglas. $\tau = -1/(\sigma - 1)$

Welfare Costs of Inflation in Units of Flex. Price Consumption (%)

- In a Cobb-Douglas economy with no steady-state distortions:



π_{ss}	Calibrated	Std NK	Ratio
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2.5	0.2743	0.0168	16.3
3.0	0.4103	0.0243	16.9
3.5	0.5815	0.0333	17.5
4.0	0.7927	0.0436	18.2

Table

Literature

- Optimal rate of inflation in monetary models
  Schmitt-Grohé and Uribe (2010), Woodford (2010)

- Welfare cost of inflation in a round-about sticky price economy
  Christiano (2015)

- Welfare cost of inflation in New Keynesian models
  Nakamura et al. (2018)

- Steady-state distortions and aggregate productivity in production networks
  Baqaee and Farhi (2020), Bigio and La'O (2020)

Model

Model: Steady State Analysis of Afrouzi and Bhattarai (2023)

- Time is continuous
- n industries indexed by $i \in [n] \equiv \{1, \ldots, n\}$
- A measure of monopolistically competitive intermediate firms in each sector
- A final good producer in each sector packages and sells a sectoral good
- Sectoral goods consumed by households and used for production
- **Objective:** Steady-state welfare comparative statics w.r.t. inflation

- Household

$$\max \int_0^\infty e^{-\rho t} U(C_t, L_t)\mathrm{d}t$$

$$\sum_{i\in[n]} P_{i,t} C_{i,t} + \dot{B}_t \leq W_t L_t + i_t B_t + T_t$$

$$C_t \equiv \Phi(C_{1,t}, \ldots, C_{n,t})$$

$$P_t \equiv \sum_{i\in[n]} P_{i,t} C_{i,t} / C_t$$

- **Household**

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- **Monetary Policy**
controls $\{M_t = P_t C_t\}_{t\geq 0}$:

$$\dot{M}_t = \pi M_t$$

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- **Monetary Policy**

controls $\{M_t = P_t C_t\}_{t \geq 0}$:

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- **Final Good** Producer

$$\max P_{i,t} Y_{i,t} - \int_0^1 P_{ij,t} Y^d_{ij,t} \mathrm{d}j \quad s.t.$$

$$Y_{i,t} = \left[\int_0^1 (Y^d_{ij,t})^{1-\sigma_i^{-1}} \mathrm{d}j \right]^{\frac{1}{1-\sigma_i^{-1}}}$$

Model–Intermediate Good Producers

- **Production**: Firm ij, $j \in [0,1]$ produces with a CRS production function

$$Y^S_{ij,t} = Z_{i,t} F_i(L_{ij,t}, X_{ij,1,t}, \ldots, X_{ij,n,t})$$

- Arbirtrary production structure with aggregate and sectoral shocks

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- **Pricing**: In sector i, i.i.d. price changes arrive at Poisson rate $\theta_i > 0$
- A firm ij that gets to change its price at time t maximizes

$$\max_{P_{ij,t}} \int_0^\infty \theta_i e^{-(\theta_i h + \int_0^h i_{t+s} \mathrm{d}s)} [\underbrace{(1-\tau_i) P_{ij,t} \mathcal{D}(P_{ij,t}/P_{i,t+h}; Y_{i,t+h})}_{\text{total revenue at time } t} - \underbrace{\mathcal{C}_i(Y^S_{ij,t+h}; \mathbf{P}_{t+h}, Z_{i,t+h})}_{\text{total cost at time } t}] \mathrm{d}h$$

$$\text{subject to} \quad Y^S_{ij,t+h} \geq \mathcal{D}(P_{ij,t}/P_{i,t+h}; Y_{i,t+h}), \quad \forall h \geq 0$$

- Heterogeneous Calvo-type price stickiness across sectors

Theoretical Results

- Cost minimization of firms with sector i implies sectoral production function:

$$Y_i = \frac{Z_i}{D_i} F_i(L_i, X_{i,1}, \ldots, X_{i,n}), \quad D_i \equiv \int_0^1 \left(\frac{P_{ij}}{P_i}\right)^{-\sigma_i} \mathrm{d}j \geq 1$$

Steady State Allocations and Sources of Inefficiencies

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- Aggregation of relative prices implies that for $x \leq x^*$, $Pr(P_{ij}/P_j \leq x) \propto x^{-\frac{\theta_i}{\pi}}$:

$$D_i = \frac{\theta_i}{\theta_i - \sigma_i \pi} \left(1 - \frac{\sigma_i - 1}{\theta_i} \pi\right)^{\frac{\sigma_i}{\sigma_i - 1}} = \exp\left\{\frac{\sigma_i}{2}\left(\frac{\pi}{\theta_i}\right)^2\right\} + \mathcal{O}(\pi^3)$$

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- Optimal pricing of firms implies

$$P_i = \frac{\sigma_i}{\sigma_i - 1} \frac{1}{1 - \tau_i} \bar{M}_i(\pi) \times MC_i(W, P_1, \ldots, P_n; Z_i)$$

Baseline Today: Fiscal policy chooses $\tau_i(\pi)$ such that $P_i = MC_i$

Welfare Cost of Inflation

- Let $C(\pi)$ and $L(\pi)$ denote steady state consumption and labor with inflation π
- Define $\Lambda(\pi)$ such that

$$U(C(\pi), L(\pi)) = U(e^{-\Lambda(\pi)}C(0), L(0))$$

- $\Lambda(\pi)$ depends on (1) changes in aggregate productivity and (2) labor stimulus

Proposition

Let $Z \equiv \frac{C}{L}$ and $\mu \equiv WL/PC$ denote agg. prod. and labor share. Then:

$$\frac{\frac{\partial}{\partial\pi}U(C,L)}{U_c \times C} = \frac{\partial}{\partial\pi}\ln(Z) + (1-\mu)\frac{\partial}{\partial\pi}\ln(L)$$

- If subsidies $\tau_i(\pi)$ are optimal or $\rho \to 0$, second term is zero.
- If $U = \ln(C) - v(L)$ then:

$$\Lambda(\pi) = \int_0^\pi \frac{\frac{\partial}{\partial\pi}U(C,L)}{U_c \times C}\mathrm{d}\pi = \ln(Z(0)) - \ln(Z(\pi))$$

- Cobb-Douglas undistorted economy

$$\ln(Z) = \sum_i \lambda_i \ln(Z_i) - \sum_i \lambda_i \ln(D_i(\pi))$$

where $(\lambda_i)_{i\in[n]} = \beta^\intercal(\mathrm{I} + \mathrm{A} + \mathrm{A}^2 + \dots)$ is sector i's Domar weight

Proposition

Let $\delta_i = \theta_i^{-1}$ denote the average duration of price spells in sector i. Then:

$$\Lambda(\pi) = \sum_i \lambda_i \ln(D_i(\pi)) = \frac{\pi^2}{2} \times \sum_i \sigma_i \lambda_i \delta_i^2 + \mathcal{O}(\pi^3)$$

Today: $\sigma_i = \sigma$

Welfare Cost of Inflation: Decomposition with $\sigma_i = \sigma$

- Standard 1 sector NK model (with roundabout production, $\lambda_i = \lambda \geq 1, \delta_i = \delta$)

$$\Lambda(\pi) = \frac{\sigma\pi^2}{2} \times \lambda \times \delta^2$$

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- Multisector NK model w/ het. price stickiness but w/o production networks:

$$\Lambda(\pi) = \frac{\sigma\pi^2}{2} \times \left(var_\beta(\delta_i) + E_\beta[\delta_i]^2\right)$$

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- Multisector NK model w/ het. price stickiness and production networks:

$$\Lambda(\pi) = \frac{\sigma\pi^2}{2} \times \left(\sum_i \lambda_i\right) \times \left(var_\lambda(\delta_i) + E_\lambda[\delta_i]^2\right)$$

Quantitative Results

Calibration

- Use the IO tables from BEA at disaggregated level (393 sectors) to construct:
  - **A**: Production expenditure shares (under Cobb-Douglas technology)
  - β: Consumption expenditure shares (under Cobb-Douglas consumption aggregator)
- θ_i : Frequency of price adjustment, from Pasten et al. (2020)
- ψ : Inverse of the Frisch elasticity of labor supply
- ρ : Discount factor
- τ : Tax
- σ_i : Elasticity of substitution across varieties

Calibration

- Consumption aggregator is a CES aggregator with elasticity of substitution ϵ

$$C_t \equiv \Phi(C_{1,t}, \ldots, C_{n,t}) = \left[\sum_{i \in [n]} \beta_i^{\epsilon^{-1}} C_{i,t}^{1-\epsilon^{-1}} \right]^{\frac{1}{1-\epsilon^{-1}}}$$

(Cobb-Douglas when $\epsilon \to 1$)

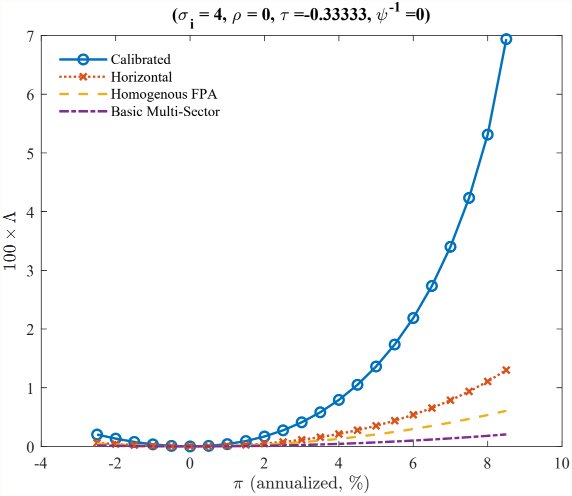
- Production function is a CES production function with elasticity of substitution η_i

$$F_i(L_{ij,t}, X_{ij,1,t}, \ldots, X_{ij,n,t}) = \left[\alpha_i^{\eta_i^{-1}} L_{i,t}^{1-\eta_i^{-1}} + \sum_{i \in [n]} a_{ij}^{\eta_i^{-1}} X_{ij,t}^{1-\eta_i^{-1}} \right]^{\frac{1}{1-\eta_i^{-1}}}$$

(Cobb-Douglas when $\eta_i \to 1$)

Model Experiments

- Start with inelastic aggregate labor supply and then endogenegize it
- First a Cobb Douglas economy and then a general CES economy
- Address how non-vertical Phillips curve interacts with (flex-price steady-state) distortions
- Various model counterfactuals
    - No production networks but heterogeneous price stickiness across sectors
    - Production networks but homogeneous price stickiness across sectors
    - No production networks and homogeneous price stickiness across sectors


(σ_i = 4, ρ = 0, τ =-0.33333, ψ^{-1} =0)
Calibrated
Horizontal
Homogenous FPA
Basic Multi-Sector
$100 \times \Lambda$
π (annualized, %)
0
1
2
3
4
5
6
7
-4
-2
0
2
4
6
8
10

Welfare Cost of Inflation at 4% Annual Inflation

- Standard NK model with average freq.:

$$\Lambda(\pi) = \frac{\sigma\pi^2}{2} \times \underbrace{\delta^2}_{=4.36^2=19} = 0.041\%$$

π_{ss}	Calibrated	Std NK	Ratio
1.0	0.0395	0.0027	14.9
1.5	0.0920	0.0060	15.3
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- Standard NK model with average dur.:

$$\Lambda(\pi) = \frac{\sigma\pi^2}{2} \times \underbrace{\delta^2}_{=7.28^2=53} = 0.12\%$$

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$$\Lambda(\pi) = \frac{\sigma\pi^2}{2} \times \underbrace{\delta^2}_{=7.28^2=53} = 0.12\%$$

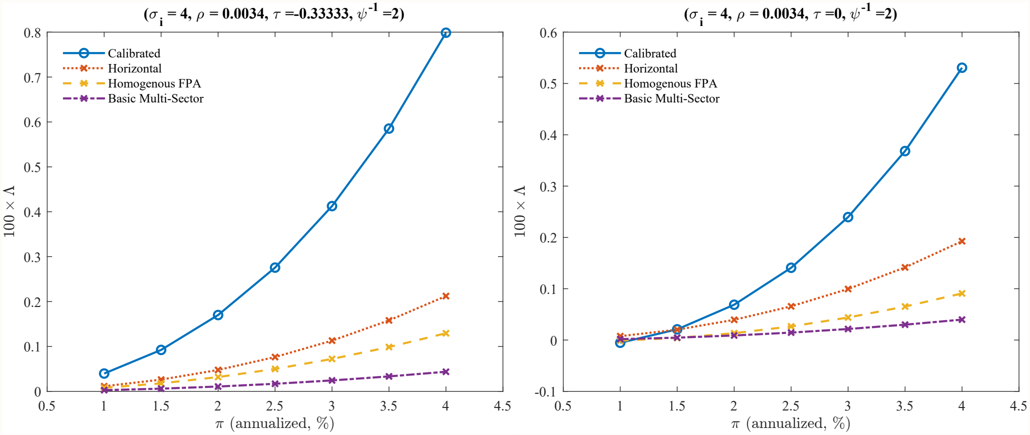
- Model w/ prod. network and het. freq.:

$$\underbrace{\Lambda(\pi) = \frac{\sigma\pi^2}{2} \times \underbrace{\left(\sum_i \lambda_i\right)}_{=4} \times \left(\underbrace{var_\lambda(\delta_i)}_{=31.83} + \underbrace{E_\lambda[\delta_i]^2}_{=62.57}\right)}_{=0.82\%}$$

Model with endogeneous labor supply

- Now move to endogenous labor supply
- Frisch elasticity of 2: $\psi^{-1} = 2$
- Welfare effects of inflation now have two sources:
    - Productivity effects
    - Labor stimulus effects
- Interacts with non-vertical long-run Phillips curve and distortions under flexible prices

Comparative Statics: $\psi^{-1} = 2$, Cobb-Douglas, $\rho = 0.0034$, $\tau = 0$

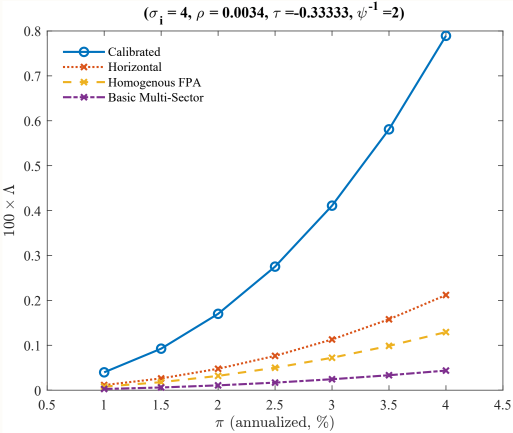


(a) Cobb-Douglas. $\tau = -1/(\sigma - 1)$

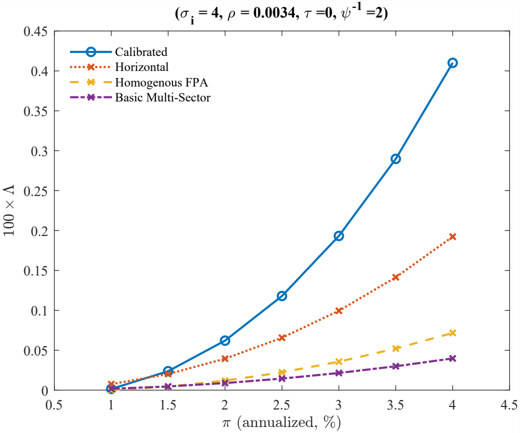
(b) Cobb-Douglas. $\tau = 0$

CES

Comparative Statics: $\psi^{-1} = 2$, CES, $\rho = 0.0034$, $\tau = 0$, $\eta_i = \epsilon = 2$



(a) CES. $\tau = -1/(\sigma - 1), \eta = \epsilon = 2$



(b) CES. $\tau = 0, \eta = \epsilon = 2$

Comparative Statics: $\psi^{-1} = 2$, CES, $\rho = 0.0034$, $\tau = 0, \eta = \epsilon = 2$

π_{SS}	Calibrated	Basic Multi-Sector	Ratio
1.0	0.001546	0.001759	0.878861
1.5	0.023775	0.004644	5.119135
2.0	0.062067	0.008889	6.982374
2.5	0.117958	0.014509	8.129850
3.0	0.193201	0.021522	8.976966
3.5	0.289802	0.029944	9.678089
4.0	0.410077	0.039794	10.304983

Conclusion

Conclusion

- Multi-sector sticky price model critical for quantitative evaluation of welfare cost of inflation
- Production networks significantly amplify welfare cost of inflation
- Future work
    - Idiosyncratic firm-level shocks
    - Generalized hazard function/Menu costs

Appendix

Let i index sector. Then, the labor share and the expenditure shares are given by

$$\alpha_i(\mathbf{p}(\pi)) = \frac{\alpha_i}{\alpha_i + \sum_{j\in[n]} a_{ij} \left(\frac{P_j}{W}\right)^{1-\eta_i}}$$

$$a_{ij}(\mathbf{p}(\pi)) = \frac{a_{ij} P_j^{1-\eta_i}}{\alpha_i W^{1-\eta_i} + \sum_{j\in[n]} a_{ij} P_j^{1-\eta_i}}$$

Marginal Cost of firms in sector i:

$$MC_i = \frac{1}{Z_i}\left[\alpha_i W^{1-\eta_i} + \sum_{j\in[n]} a_{ij} P_j^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}}$$

Let π be the steady state inflation rate. Then the sector i markup (P_i/MC_i) is given by

$$\mu_i(\pi) \equiv \frac{\sigma_i}{\sigma_i - 1} \frac{1}{(1-\tau_i)} \frac{\rho + \theta_i - (\sigma_i - 1)\pi}{\rho + \theta_i - \sigma_i \pi} \left[1 - \frac{(\sigma_i - 1)\pi}{\theta_i}\right]^{\frac{1}{\sigma_i - 1}}$$

The equilibrium sector prices $(P_i)_{i\in[n]}$ satisfy

$$(\frac{P_i}{W}) = \frac{\mu_i(\pi)}{Z_i} \left(\alpha_i + \sum_{j\in[n]} a_{ij} (\frac{P_j}{W})^{1-\eta_i}\right)^{\frac{1}{1-\eta_i}}$$

Let π be the steady state inflation rate. Then, the price dispersion in sector i, D_i is given by

$$D_i(\pi) = \frac{\theta_i}{\theta_i - \sigma_i \pi} \left[1 - \frac{(\sigma_i - 1)\pi}{\theta_i} \right]^{\frac{\sigma_i}{\sigma_i - 1}}$$

The household's consumption share of sector i is given by

$$\beta_i(\mathbf{p}(\pi)) = \frac{\beta_i(P_i/W)^{1-\epsilon}}{\sum_{j\in[n]} \beta_j(P_i/W)^{1-\epsilon}}$$

Aggregate price index

$$P(\pi) \equiv \left[\sum_{i\in[n]} \beta_i P_i^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

Let $\boldsymbol{\mathcal{M}} \equiv \text{diag}\left({}^{\mu_i(\pi)}/_{D_i(\pi)}\right)$

$$Z(\pi) \equiv \frac{C}{L} = \frac{1}{\left[\sum_{i \in [n]} \beta_i \left(\frac{P_i}{W}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}} \times \frac{1}{\mathbf{1}'(\mathbf{I} - \mathbf{A}(\mathbf{p}(\pi))')(\boldsymbol{\mathcal{M}} - \mathbf{A}(\mathbf{p}(\pi))')^{-1}\boldsymbol{\beta}(\mathbf{p}(\pi))}$$

Welfare a la Lucas

$$U(C(\pi), L(\pi)) = U(e^{-\Lambda}C(0), L(0))$$

In each economy: $C(\pi) = Z(\pi)L(\pi)$, $C(0) = Z(0)L(0)$, with $U(C, L) = \ln(C) - \frac{L^{1+\frac{1}{\psi-1}}}{1+\frac{1}{\psi-1}}$.

Also,

$$\frac{WL}{PC} = \boldsymbol{\alpha}'\boldsymbol{\mathcal{M}}^{-1}\boldsymbol{\lambda} = \mathbf{1}'(\mathbf{I} - \mathbf{A}')(\boldsymbol{\mathcal{M}} - \mathbf{A}')^{-1}\boldsymbol{\beta} \text{ [Labor share]}$$

$$\frac{W}{P} = CL^{\frac{1}{\psi-1}} \text{ [Optimal intratemporal condition]}$$

Remark: Flex price equilibrium = Zero SS inflation equilibrium

$$L(\pi) = \boldsymbol{\alpha}'(\pi)\boldsymbol{\mathcal{M}}^{-1}(\pi)\boldsymbol{\lambda}(\pi)$$

$$L(0) = \boldsymbol{\alpha}'(0)\boldsymbol{\mathcal{M}}^{-1}(0)\boldsymbol{\lambda}(0)$$

We calculate Λ such that

$$\Lambda + \ln(Z(\pi)) + \ln(L(\pi)) - \frac{L(\pi)^{1+\frac{1}{\psi-1}}}{1+\frac{1}{\psi-1}} = \ln(Z(0)) + \ln(L(0)) - \frac{L(0)^{1+\frac{1}{\psi-1}}}{1+\frac{1}{\psi-1}}$$

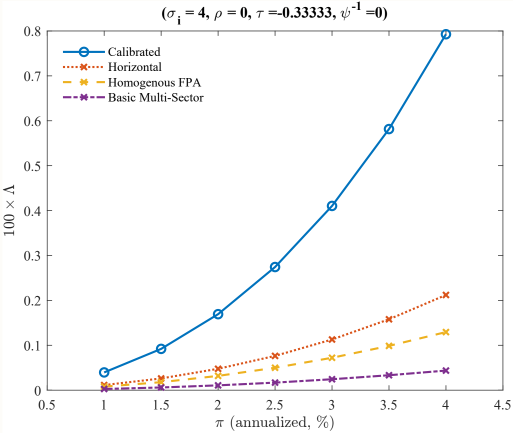
That is, we find

$$\Lambda = \ln\left(\frac{Z(0)}{Z(\pi)}\right) + \frac{1}{1+\frac{1}{\psi-1}}\ln\left(\frac{\boldsymbol{\alpha}'(0)\boldsymbol{\mathcal{M}}^{-1}(0)\boldsymbol{\lambda}(0)}{\boldsymbol{\alpha}'(\pi)\boldsymbol{\mathcal{M}}^{-1}(\pi)\boldsymbol{\lambda}(\pi)}\right) + \frac{\boldsymbol{\alpha}'(\pi)\boldsymbol{\mathcal{M}}^{-1}(\pi)\boldsymbol{\lambda}(\boldsymbol{\pi})}{1+\frac{1}{\psi-1}} - \frac{\boldsymbol{\alpha}'(0)\boldsymbol{\mathcal{M}}^{-1}(0)\boldsymbol{\lambda}(0)}{1+\frac{1}{\psi-1}}$$

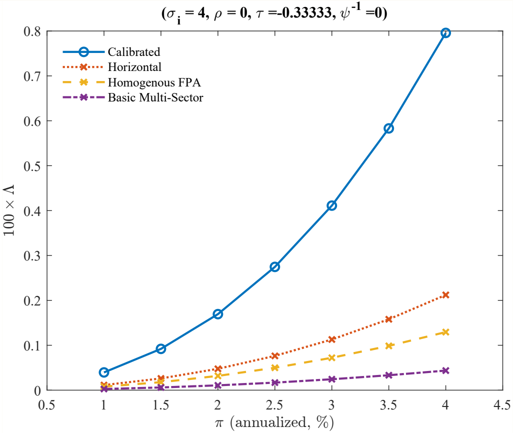
Economy with complementarity

- Now consider a CES economy
- Complementarity in demand and production: $\eta = \epsilon = 0.8$

Comparative Statics: $\psi^{-1} = 0$, CES, $\eta = \epsilon = 0.8$, $\rho = 0$, $\tau = -1/(\sigma - 1)$

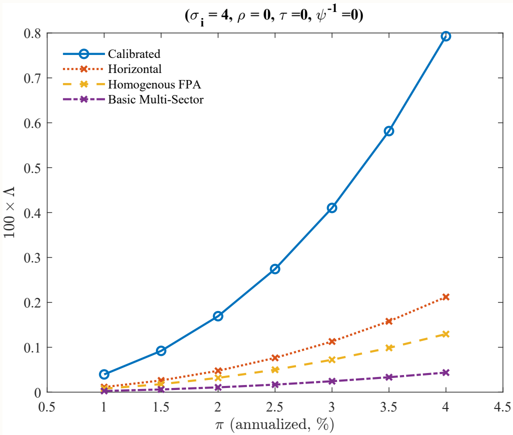


(a) Cobb-Douglas. $\tau = -1/(\sigma - 1)$

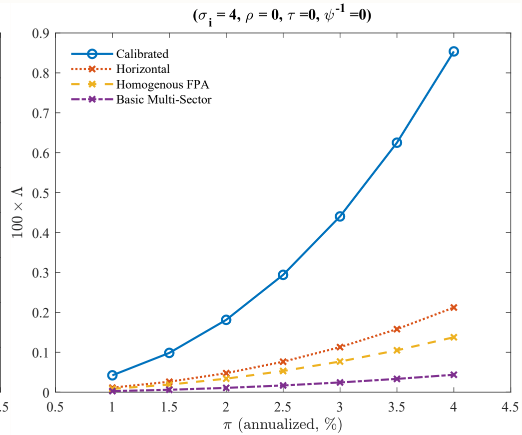


(b) CES. $\tau = -1/(\sigma - 1)$

Comparative Statics: $\psi^{-1} = 0$, CES, $\eta = \epsilon = 0.8$, $\rho = 0$, $\tau = 0$



(a) Cobb-Douglas. $\tau = 0$

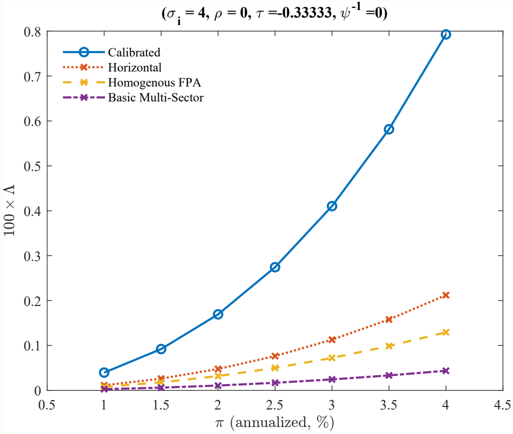


(b) CES. $\tau = 0$

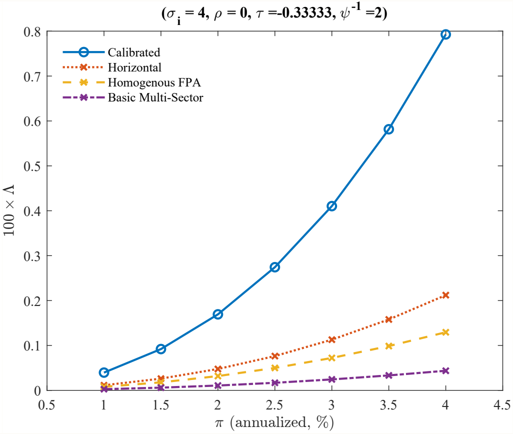
Model with endogeneous labor supply

- Now move to endogenous labor supply
- Frisch elasticity of 2: $\psi^{-1} = 2$
- Welfare effects of inflation now have two sources:
    - Productivity effects
    - Labor stimulus effects

Comparative Statics: $\psi^{-1} = 2$, Cobb-Douglas, $\rho = 0$, $\tau = -1/(\sigma - 1)$



(a) Cobb-Douglas. $\psi^{-1} = 0$



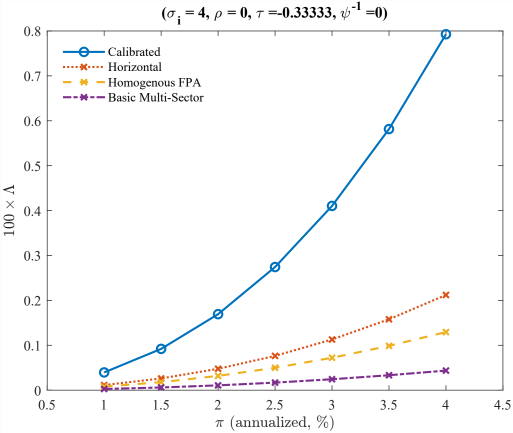
(b) Cobb-Douglas. $\psi^{-1} = 2$

$\tau = 0$

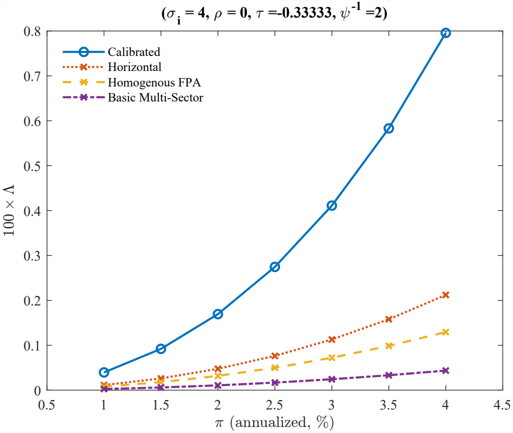
Economy with complementarity

- Now consider a CES economy
- Complementarity in demand and production: $\eta = \epsilon = 0.8$

Comparative Statics: $\psi^{-1} = 2$, CES, $\eta = \epsilon = 0.8$, $\rho = 0$, $\tau = -1/(\sigma - 1)$

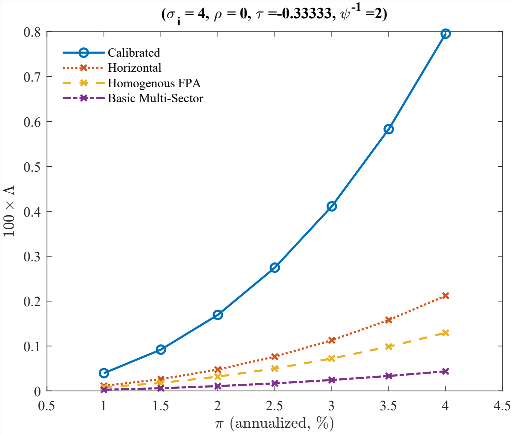


(a) Cobb-Douglas. $\psi^{-1} = 0$

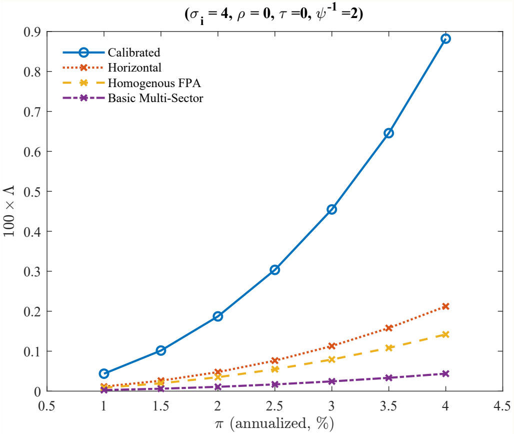


(b) CES. $\psi^{-1} = 2$

Comparative Statics: $\psi^{-1} = 2$, CES, $\eta = \epsilon = 0.8$, $\rho = 0$, $\tau = 0$

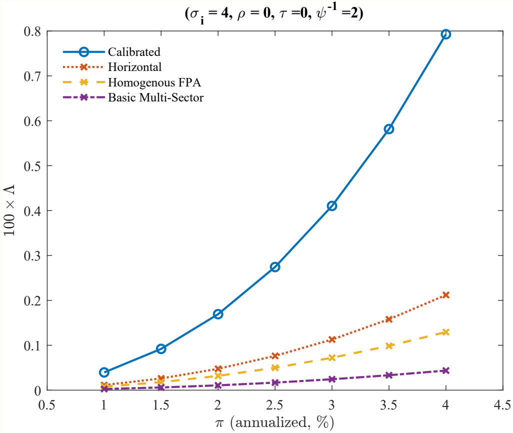


(a) CES. $\tau = -1/(\sigma - 1)$

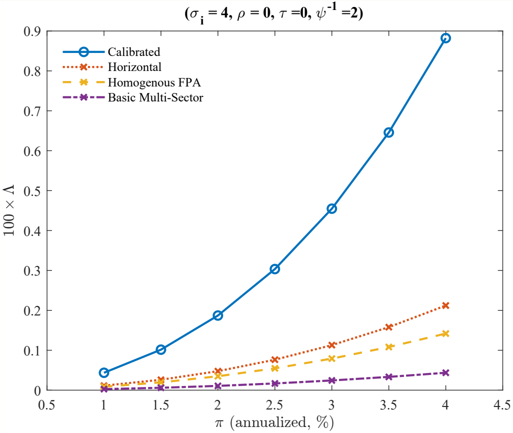


(b) CES. $\tau = 0$

Comparative Statics: $\psi^{-1} = 2$, Cobb-Douglas $\times$ CES, $\rho = 0$, $\tau = 0$



(a) Cobb-Douglas. $\tau = 0$

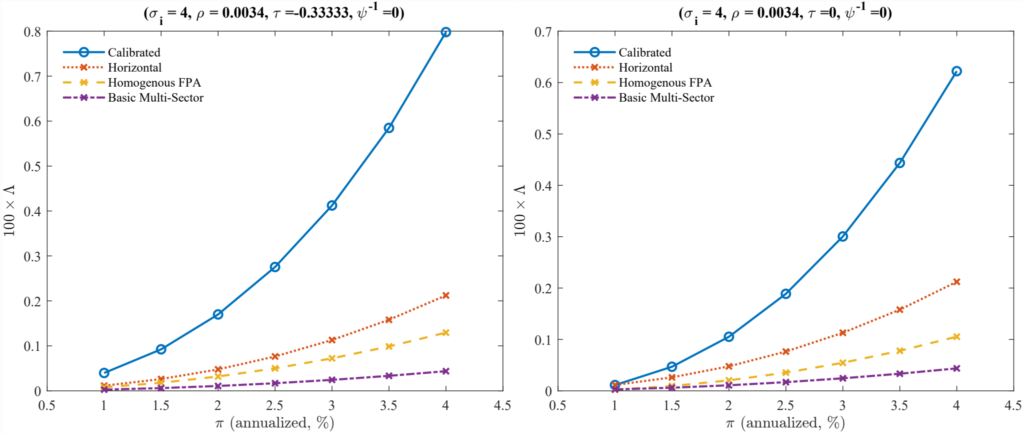


(b) CES. $\tau = 0$

Non-vertical Philips curve

- If the long-run Phillips curve is not vertical, some quantitaitve differences in results
- But qualitiative differences come about when comparing disorted vs. undisorted economies (under flex-prices)

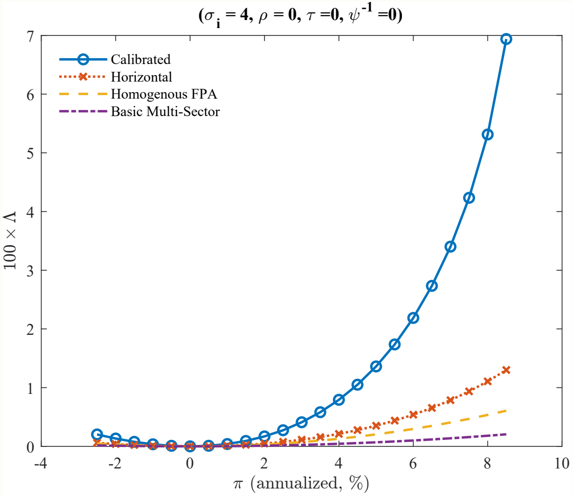
Comparative Statics: $\psi^{-1} = 0$, Cobb-Douglas, $\rho = 0.0034$, $\tau = -1/(\sigma - 1)$



(a) Cobb-Douglas. $\tau = -1/(\sigma - 1)$

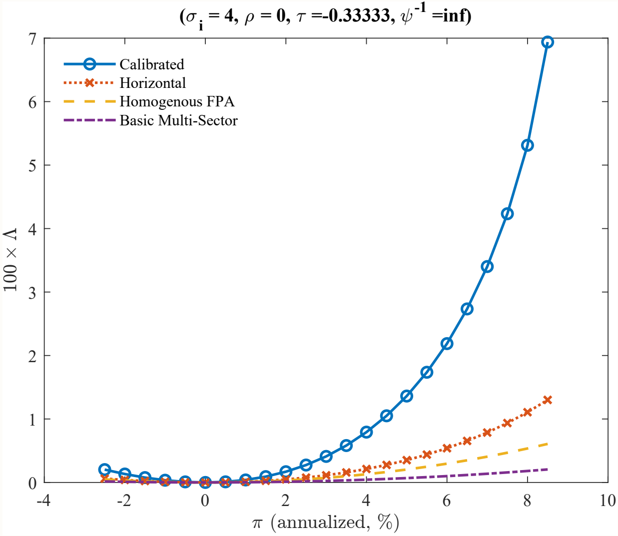
(b) Cobb-Douglas. $\tau = 0$

Comparative Statics: $\psi^{-1} = 0$, Cobb-Douglas, $\rho = 0$, $\tau = 0$



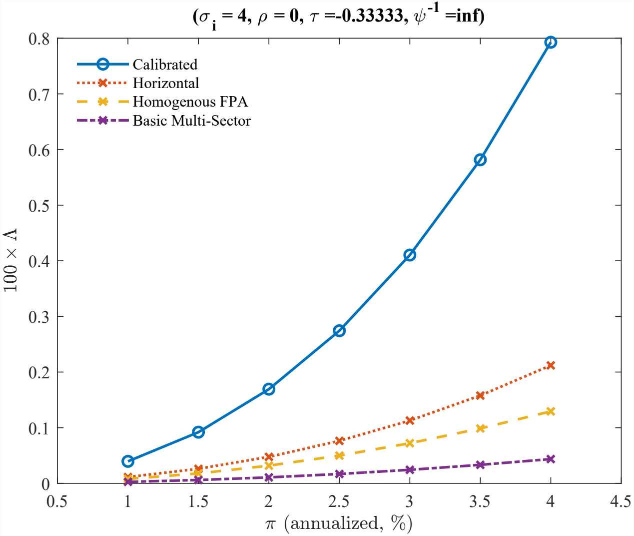
Back

Comparative Statics: $\psi = 0$, Cobb-Douglas, $\rho = 0$, $\tau = -1/(\sigma - 1)$

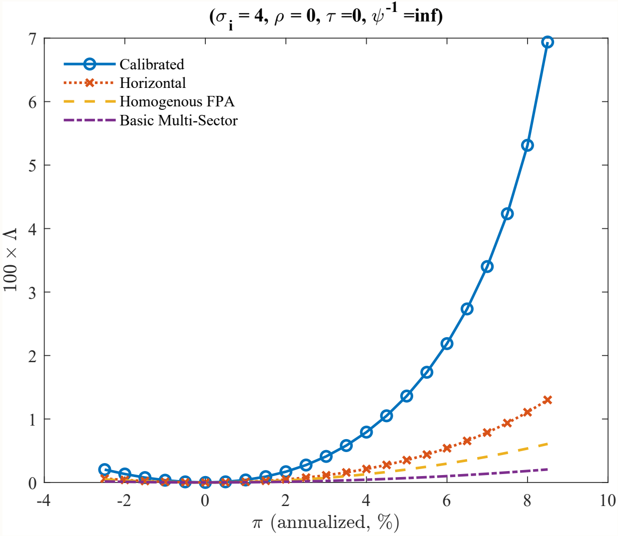


Back

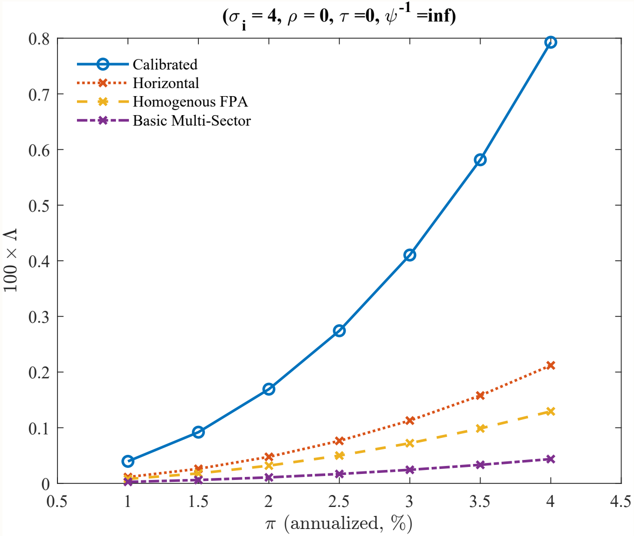
Comparative Statics: $\psi = 0$, Cobb-Douglas, $\rho = 0$, $\tau = -1/(\sigma - 1)$



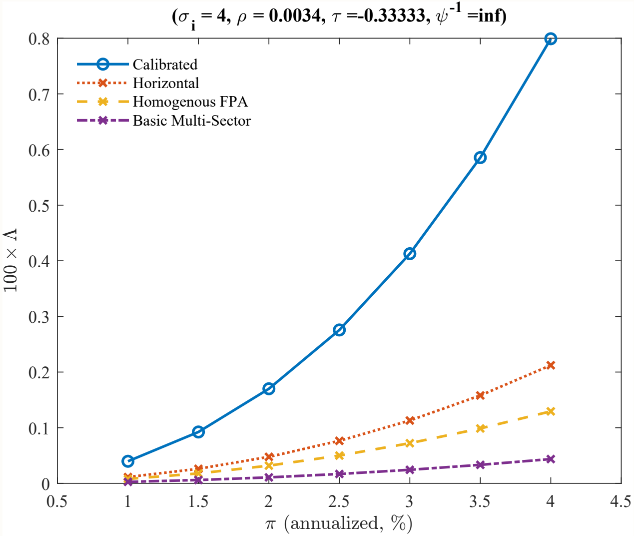
Comparative Statics: $\psi = 0$, Cobb-Douglas, $\rho = 0$, $\tau = 0$



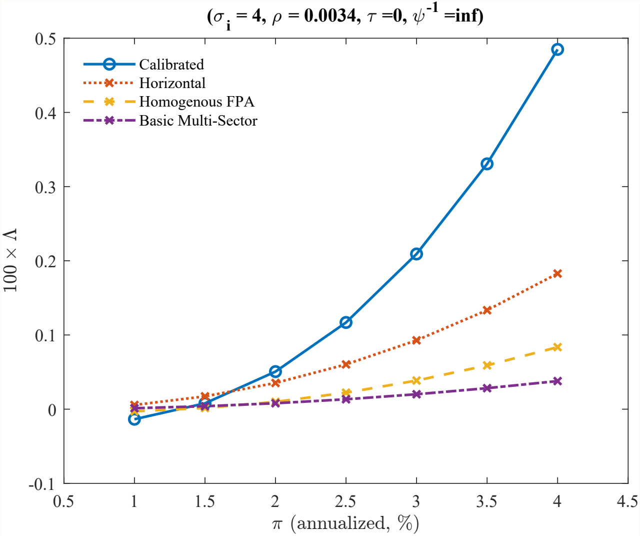
Comparative Statics: $\psi = 0$, Cobb-Douglas, $\rho = 0$, $\tau = 0$



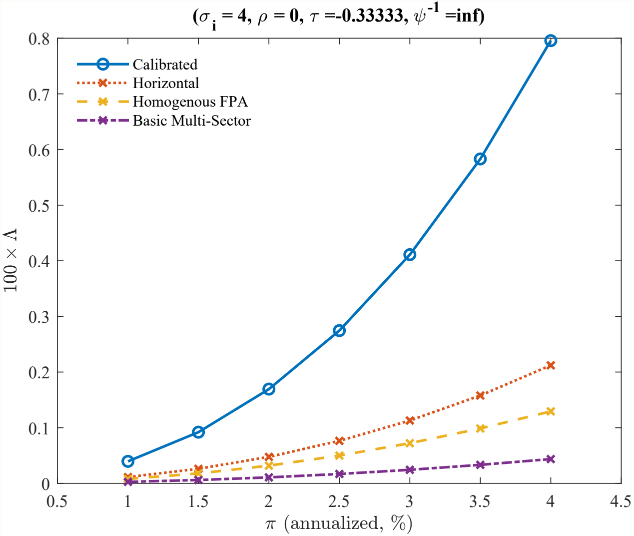
Comparative Statics: $\psi = 0$, Cobb-Douglas, $\rho = 0.0034$, $\tau = -1/(\sigma - 1)$



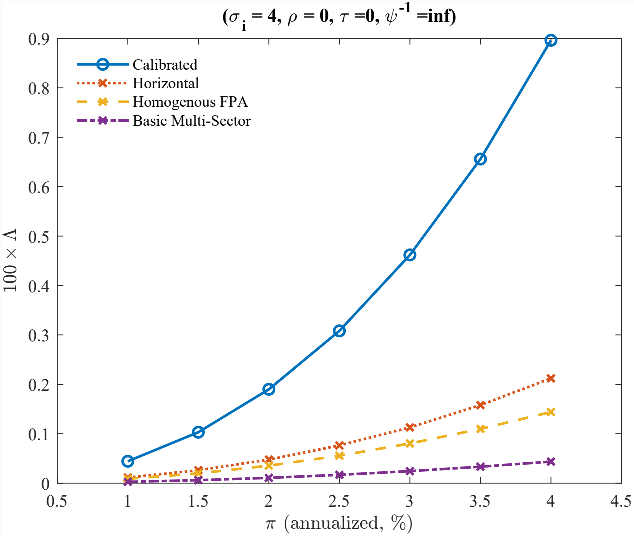
Comparative Statics: $\psi = 0$, Cobb-Douglas, $\rho = 0.0034$, $\tau = 0$



Comparative Statics: $\psi = 0$, CES, $\eta = \epsilon = 0.8$, $\rho = 0$, $\tau = -1/(\sigma - 1)$

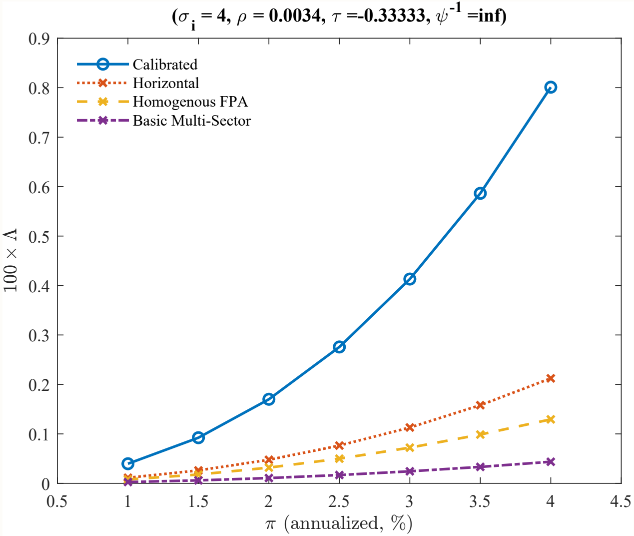


Comparative Statics: $\psi = 0$, CES, $\eta = \epsilon = 0.8$, $\rho = 0$, $\tau = 0$

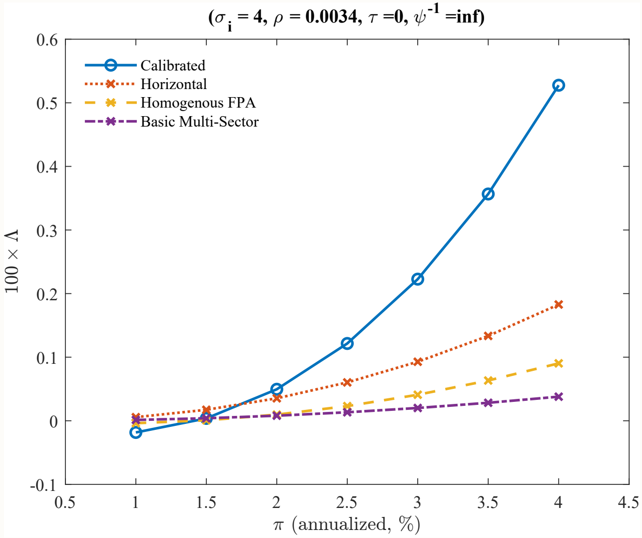


Comparative Statics: $\psi = 0$, CES, $\eta = \epsilon = 0.8$, $\rho = 0.0034$, $\tau = -1/(\sigma - 1)$

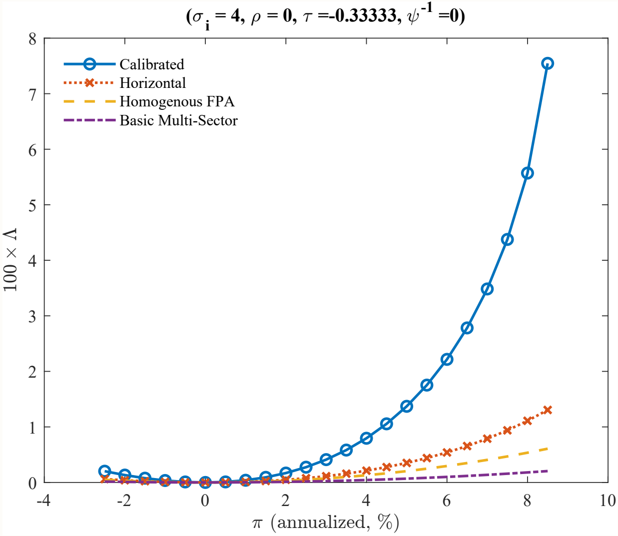
Comparative Statics: $\psi = 0$, CES, $\eta = \epsilon = 0.8$, $\rho = 0.0034$, $\tau = -1/(\sigma - 1)$



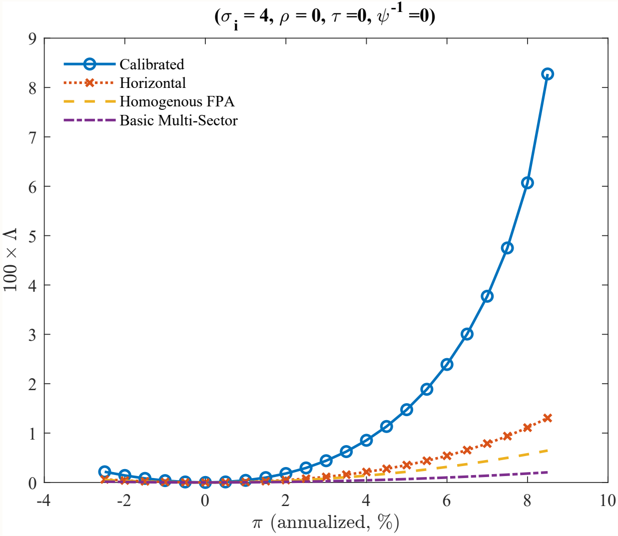
Comparative Statics: $\psi = 0$, CES, $\eta = \epsilon = 0.8$, $\rho = 0.0034$, $\tau = 0$



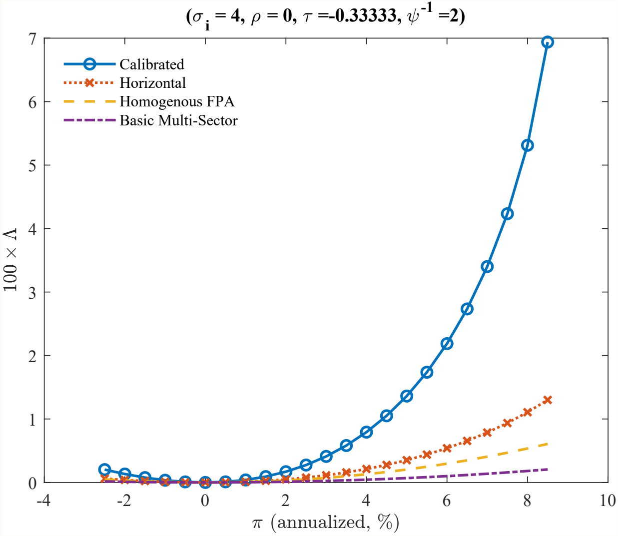
Comparative Statics: $\psi^{-1} = 0$, CES, $\eta = \epsilon = 0.8$, $\rho = 0$, $\tau = -1/(\sigma - 1)$



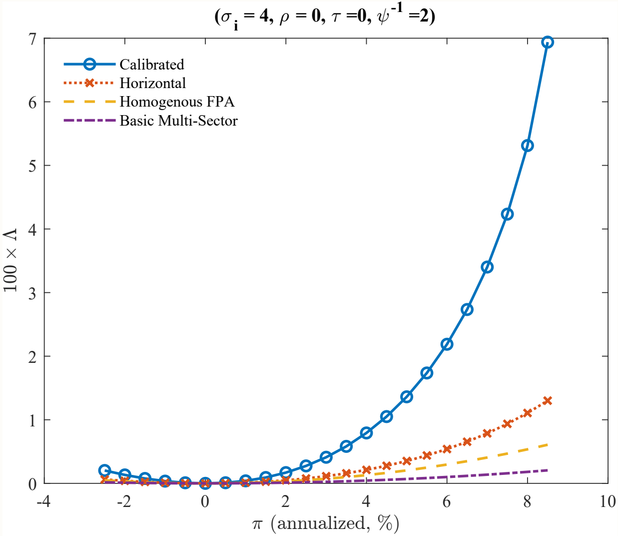
Comparative Statics: $\psi^{-1} = 0$, CES, $\eta = \epsilon = 0.8$, $\rho = 0$, $\tau = 0$



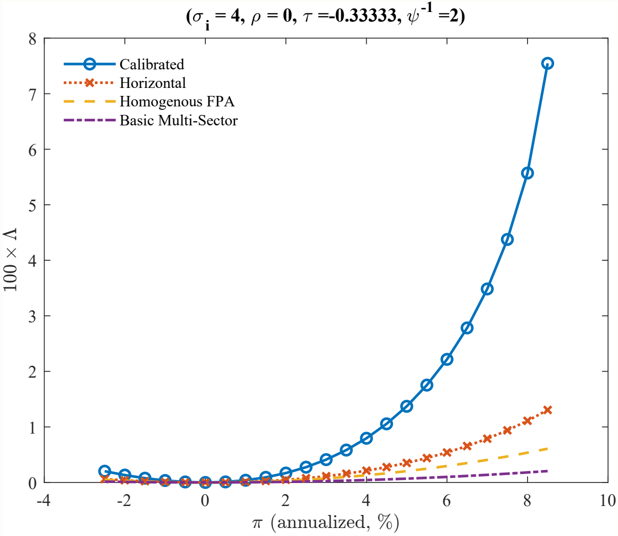
COMPARATIVE STATICS: $\psi^{-1} = 2$, COBB-DOUGLAS, $\rho = 0$, $\tau = -1/(\sigma - 1)$



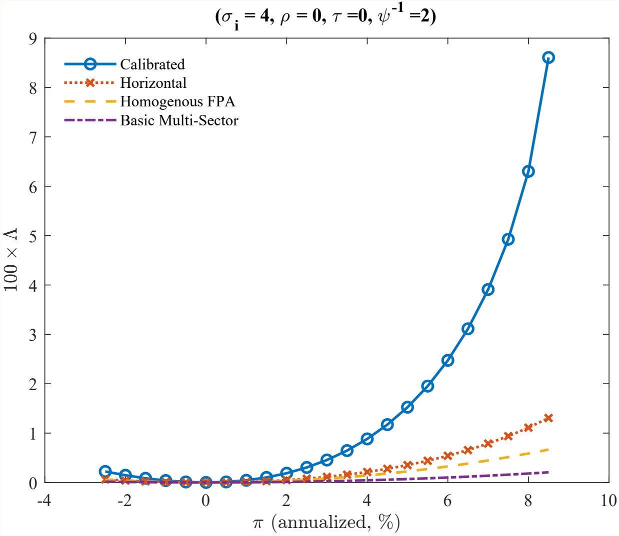
Comparative Statics: $\psi^{-1} = 2$, Cobb-Douglas, $\rho = 0$, $\tau = 0$



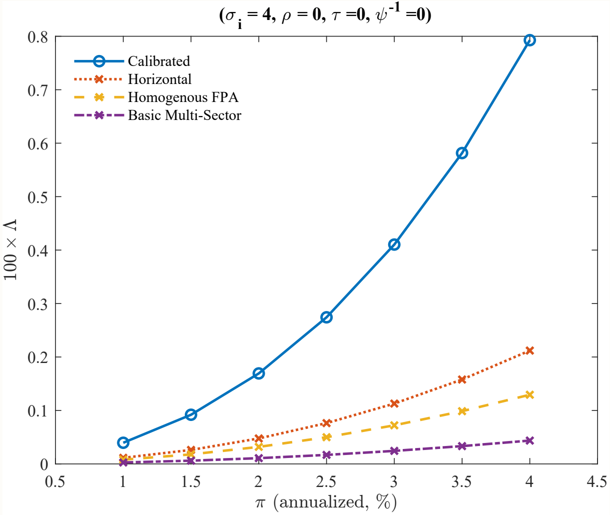
Comparative Statics: $\psi^{-1} = 2$, CES, $\eta = \epsilon = 0.8$, $\rho = 0$, $\tau = -1/(\sigma - 1)$



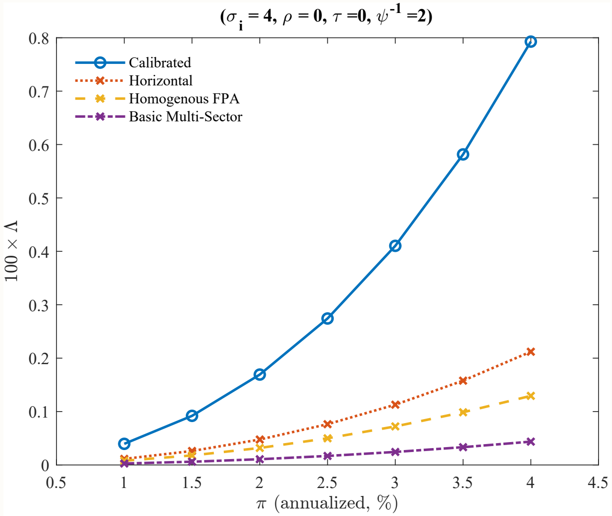
Comparative Statics: $\psi^{-1} = 2$, CES, $\eta = \epsilon = 0.8$, $\rho = 0$, $\tau = 0$



Comparative Statics: $\psi^{-1} = 0$, Cobb-Douglas, $\rho = 0$, $\tau = 0$



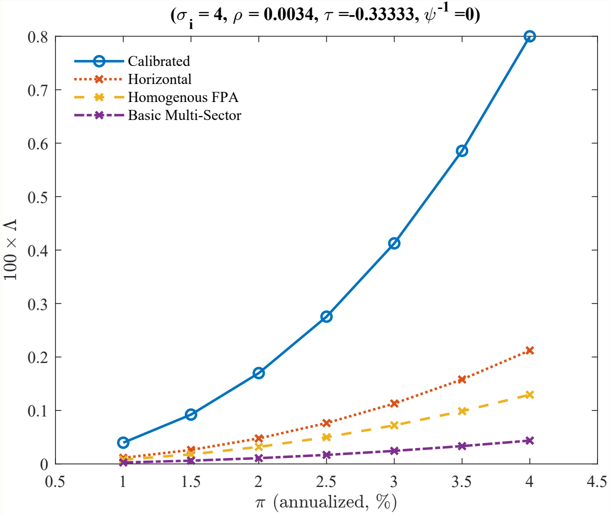
Back



(σ_i = 4, ρ = 0, τ =0, ψ^{-1} =2)
Calibrated
Horizontal
Homogenous FPA
Basic Multi-Sector
100 × Λ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.5
1
1.5
2
2.5
3
3.5
4
4.5
π (annualized, %)

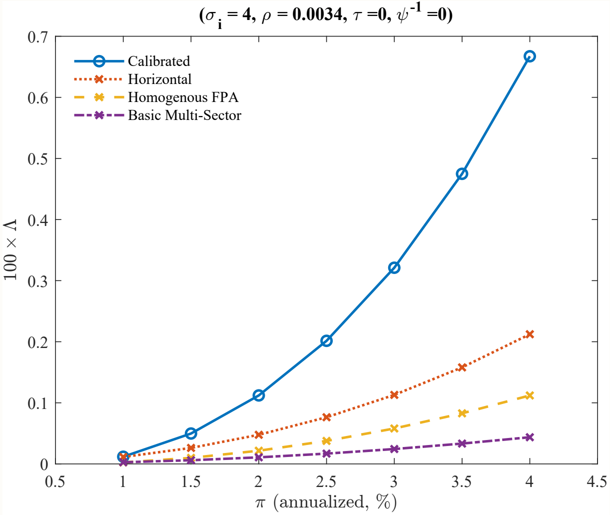
Back

Comparative Statics: $\psi^{-1} = 0$, CES, $\eta = \epsilon = 0.8$, $\rho = 0.0034$, $\tau = -1/(\sigma - 1)$



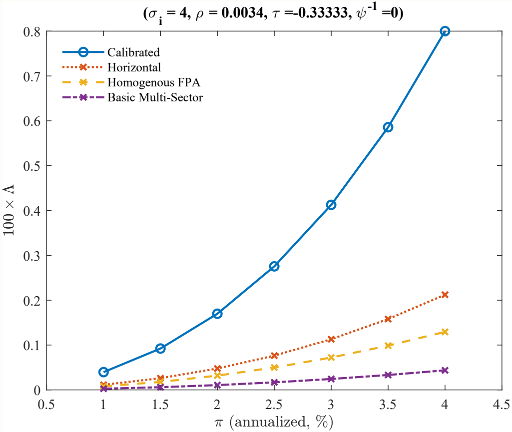
Back

Comparative Statics: $\psi^{-1} = 0$, CES, $\eta = \epsilon = 0.8$, $\rho = 0.0034$, $\tau = 0$

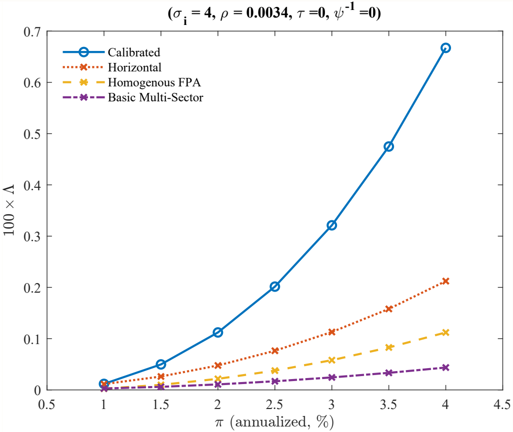


Back

Comparative Statics: $\psi^{-1} = 0$, Cobb-Douglas, $\rho = 0.0034$, $\tau = -1/(\sigma - 1)$



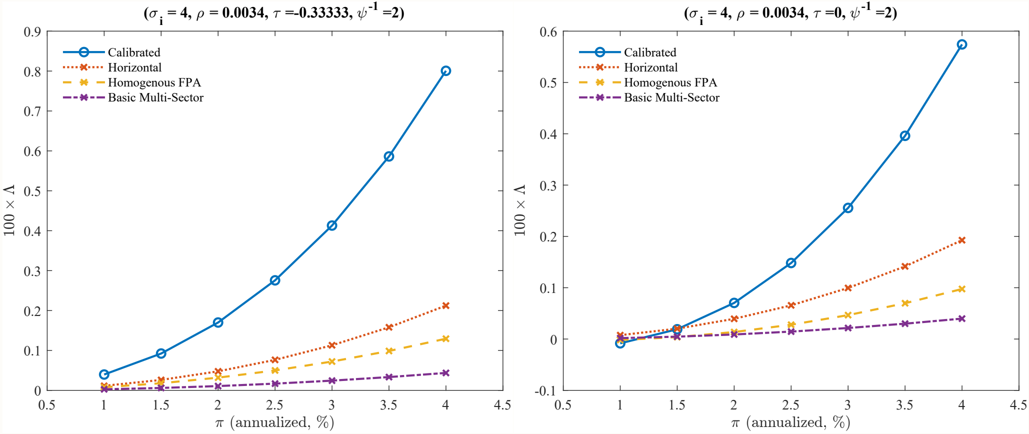
(a) CES. $\tau = -1/(\sigma - 1)$



(b) CES. $\tau = 0$

Back

Comparative Statics: $\psi^{-1} = 2$, CES, $\rho = 0.0034$, $\tau = 0$



(a) CES. $\tau = -1/(\sigma - 1)$

(b) CES. $\tau = 0$

Back

Comparative Statics: $\psi^{-1} = 0$, Cobb-Douglas, $\rho = 0$, $\tau = -1/(\sigma - 1)$

π_{SS}	Calibrated	Basic Multi-Sector	Ratio
1.0	0.039539	0.002662	14.852318
1.5	0.091962	0.006014	15.292148
2.0	0.169256	0.010734	15.768539
2.5	0.274258	0.016839	16.286762
3.0	0.410333	0.024347	16.853239
3.5	0.581520	0.033276	17.475897
4.0	0.792737	0.043642	18.164680

Back

Comparative Statics: $\psi^{-1} = 0$, Cobb-Douglas, $\rho = 0.0034$, $\tau = 0$

π_{SS}	Calibrated	Basic Multi-Sector	Ratio
1.0	0.011167	0.002662	4.194599
1.5	0.046810	0.006014	7.784051
2.0	0.105196	0.010734	9.800472
2.5	0.188767	0.016839	11.209902
3.0	0.300397	0.024347	12.337974
3.5	0.443504	0.033276	13.328234
4.0	0.622200	0.043642	14.257033

Comparative Statics: $\psi^{-1} = 2$, CES, $\rho = 0.0034$, $\tau = -1/(\sigma - 1), \eta = \epsilon = 2$

π_{SS}	Calibrated	Basic Multi-Sector	Ratio
1.0	0.039758	0.002663	14.932468
1.5	0.092440	0.006014	15.370534
2.0	0.170020	0.010735	15.838630
2.5	0.275160	0.016840	16.339925
3.0	0.410950	0.024348	16.878446
3.5	0.580918	0.033277	17.457177
4.0	0.789181	0.043643	18.082539

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